

NOTE ON THE NEWCOMB OPERATORS USED IN THE DEVELOPMENT OF THE PERTURBATIVE FUNCTION.

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(Received February 27, 1913.)

(Read April 16, 1913.)

In my paper on the algebraical development of the elliptic perturbative function (TRANSACTIONS, vol. ii., part 3, pp. 301-317, 1911) certain recurrence formulae were given by which the Newcomb operators involved could be built up from those of a lower order. As therein remarked, Newcomb and Chessin had already done the same, but the advantage in my method was the brevity of the calculations and the disclosure of the recurrence-law which permitted the indefinite extension for primary and secondary terms. I now add two tables which will include all the most important tertiary terms (namely, all those under the 6th order of the excentricities).

Herr G. v. Zeipel has recently published a paper on these operators (Arkiv för Matematik, Astronomi och Fysik, Stockholm, Band 8, No. 19, 1912), which gives a very simple analytical solution of the problem. He has found—

$$4Q\Pi_q^q = (-2D + 4i)\Pi_{q-1}^{q-1} + (-D + i)\Pi_{q-2}^{q-2}$$

(i)
(i+1)
(i+2)

and expressions of the same nature for Π_q^{q+2} and Π_q^{q+4} .

If explicit developments in terms of D and i are required, these are the most convenient expressions yet derived, but in practical use they suffer from a disadvantage which the older recurrence formulae avoid. Herr v. Zeipel's formulae only build up the operators, and it is therefore necessary to introduce the i 's as has been done, whilst the others build up both operator and its object. These operators, when numerical applications are made, must be attached to the $a'A$, of the perturbative function.

TABLE IV.A.

TERTIARY TERMS.

	$\Pi_{0,n}^{0,n'}$	$2\Pi_{1,n}^{1,n'}$	$8\Pi_{2,n}^{2,n'}$	$48\Pi_{3,n}^{3,n'}$	$384\Pi_{4,n}^{4,n'}$	$3840\Pi_{5,n}^{5,n'}$
$8\Pi_{-2,n}^{-2,n'}$	$+2(8i^2 + 5i + 0)$	$+8(i - 0)$	$+1$			
$16\Pi_{-1,n}^{-1,n'}$	$-8(3i^2 + i + 0)$	$+2(8i^2 - 14i + 2)$	$+8(i - 1)$	$+1$		
$64\Pi_{0,n}^{0,n'}$	$-2(7i^2 + i + 0)$	$-8(6i^2 - 7i + 2)$	$+2(8i^2 - 33i + 24)$	$+8(i - 2)$	$+1$	
$384\Pi_{1,n}^{1,n'}$	$-8(7i^2 + i + 0)$	$-2(21i^2 + 9i + 0)$	$-8(9i^2 - 24i + 17)$	$+2(8i^2 - 52i + 66)$	$+8(i - 3)$	$+1$

TABLE IV.B.

TERTIARY TERMS.

For $\Pi_{m,q}^{m',q'}$. Interchange m and n , and replace i by $-i$.N.B.—The i^2 can be replaced by D quantities, thus—

$$8\Pi_{-2,n}^{-2,n'} = 8\Pi_{-2,n}^{1,n'} + 4D2\Pi_{1,n}^{1,n'} + 2(5i + 2D)\Pi_{0,n}^{0,n'}$$

In other words, although we can with Herr v. Zeipel write symbolically (putting $q = 2$)—

$$8\Pi_{(i)}^2 = (-2D + 4i)\Pi_{(i+1)}^1 + (-D + i)\Pi_{(i+2)}^0$$

we cannot write—

$$8\Pi_i^2 a' A_i = (-2D + 4i)\Pi_i^1 a' A_{i+1} + (-D + i)\Pi_i^0 a' A_{i+2}$$

although we can write—

$$8\Pi_i^2 a' A_i = (-2D + 3 - 2i)\Pi_i^1 a' A_i + i\Pi_i^0 a' A_i$$

as given in my paper above referred to.

In a methodical application to the theory of the motion of any planet, we require $\Pi_{(i)}^q a' A_i$, then $\Pi_{(i)}^1 a' A_i$, and so on, and once in possession of these, we proceed to $\Pi_{(i)}^2 a' A_i$, &c. This is the great advantage of the recurrence formulae which keep to $\Pi_{(i)}^{q+2m}$, in which the suffix i remains the same. It would be very nice to have expressions in which $\Pi_{(i+j)}^{q+2m}$ depended on numbers inferior to $i+j$, but so far no general expression which would facilitate numerical work has been offered. A type of such an expression is—

$$4\Pi_{(i+2)}^0 = 4\Pi_{(i)}^2 - (i+2)(3i+5)a' A_{i+2} + i(3i-1)a' A_i^*$$

in which Π_i^2 for $i+2$ is built up from the value for i plus simple multiples of $a' A_i$ and $a' A_{i+2}$.

JOHANNESBURG,

February 21, 1913.